Fiscal Rules and Fiscal Policy: A Quantitative Analysis *

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Abstract

This paper provides a quantitative analysis of the optimal design of deficit limits under imperfect enforcement. Utilizing data on primary balances and fiscal rules from 2000 to 2021, I document two critical patterns in low-income countries : the persistence of primary deficits and frequent instances of noncompliance with fiscal rules. To elucidate the role of noncompliance in shaping optimal deficit limits, I employ a tractable fiscal policy model to conduct a quantitative exercise. The model incorporates a key assumption that the cost of noncompliance with a deficit limit rises as borrowing increases. Analytical results demonstrate that the optimal deficit limit is determined by the magnitude of sanctions associated with violations of fiscal rules. Calibration for Sub-Saharan African (SSA) economies suggests that an optimal deficit limit ranges from 5.12% to 9.5% of GDP, while the sanctions vary between 0.33% and 2.7% of GDP. These findings contribute to the broader discourse on the reform of fiscal rules in SSA countries, offering evidence-based insights into the trade-offs between fiscal flexibility and the enforcement of compliance mechanisms.

Keywords : Optimal policy, fiscal rule, sanctions, deficit limit, noncompliance.

JEL CLASSIFICATION : D82, E02, E62, H62

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1 Introduction

Governments implement fiscal rules to impose discipline and structure on their fiscal policies, ensuring long-term sustainability and preventing excessive deficits or debt accumulation. [1] A fiscal rule which can be established at the national and regional levels—particularly when participating in an economic union—or both. As governments worldwide often face fiscal pressures to overspend—whether due to political, social, or economic demands—adopting fiscal rules has become increasingly prevalent. According to the International Monetary Fund (IMF), over 100 countries have implemented at least one rule-based fiscal framework since the end of 2021. [2] The choice of the fiscal rules type is generally based on ad hoc criteria rather than theoretical considerations. Furthermore, fiscal rules are often imperfectly enforced, particularly in developing countries, where compliance with fiscal rule limits tends to be significantly lower than in advanced economies. [3] In these environments, it has been shown theoretically that the optimal fiscal rule takes the form of a deficit limit (Amador, Werning, and Angeletos (2006) ; Halac and Yared, 2022). However, the approaches commonly used in the literature have been mainly theoretical, often lacking empirical validation. This paper presents a quantitative analysis of the optimal deficit limit in an environment where enforcement of the limit is imperfect.

In the first part of the paper, I highlight three key motivational facts in Section 2. First, despite the significant debt relief provided to developing countries under the Heavily Indebted Poor Countries (HIPC) initiative since 1996, these economies continue to experience a persistent primary deficit on average. The HIPC initiative was designed to provide 100 percent debt relief on eligible debts through the IMF, World Bank, and the African Development Fund. While many Sub-Saharan African countries benefited from this program in the late 2000s, their primary deficit remains above 3% of GDP on average. Second, compliance with fiscal rules remains low in Sub-Saharan African (SSA) countries. In nations characterized by weak institutions, political pressures, and short-

1. A fiscal rule is defined as a numerical limit on a government's expenditure, revenue, debt, and primary deficit. A fiscal rule is tailored to address specific macroeconomic conditions and fiscal challenges within the country or the region.

2. For more details on the fiscal rules, see Davoodi et al. (2022).

3. see Davoodi et al. (2022) for more on the compliance and enforceability of the fiscal rules across countries.

term policy goals often lead to deviations from the established fiscal limits. [4] Third, a positive correlation exists between the primary deficit and compliance with a deficit limit in Sub-Saharan African (SSA) countries. Historically, countries with lower compliance with fiscal rules tend to experience higher levels of borrowing. This suggests that non-compliance with fiscal limits often leads to increased borrowing, which in turn exacerbates fiscal imbalances.

To quantify the optimal deficit limit in an environment where enforcement is imperfect, I employ a tractable fiscal policy model, building upon the seminal work of Halac and Yared (2022) that features deficit-biased small open economies and sanctions. Policy myopia can arise from political turnover, as each new government tends to prioritize short-term solutions aimed at addressing immediate issues, often at the expense of long-term policy planning. This short-term focus is driven by the need to secure political support and demonstrate quick results, which can lead to a lack of continuity in policy initiatives. As a result, the political friction could explain the tendency of a government to overspend when it holds power. In my approach, I assume that each government faces fiscal pressures and makes the borrowing decision. This feature allows the government to have the flexibility to react when economic shocks hit the economy. On the other hand, society imposes increasing sanctions on the government as the frequency of fiscal rule violations rises, aiming to discipline the government and ensure compliance with fiscal constraints. To assess the fiscal needs of each government in response to such pressures, I employ a non-parametric approach. This approach allows for the inference of the fiscal needs of each government from shocks to its revenue.

The sanctions are designed to effectively disincentive the government from overborrowing and reduce the frequency of rule non-compliance with a fiscal rule in fine. For instance, the Excessive Deficit Procedure of the Stability and Growth Pact ensures that the EU Member States with planned or actual budget deficits exceeding 3% of GDP in principle end up receiving binding recommendations from the European Council on the annual fiscal adjustment (usually defined in terms of the structural budget balance and fines starting at 0.2% of GDP). [5] In contrast, Sub-Saharan African (SSA) countries *lack a clear definition* of the sanctions within their fiscal policy frameworks. Given that

4. See Dovis and Kirpalani (2020) for their contribution to the relationship between primary balance and reputation.

5. see Diaz Kalan, F, M Popescu and J Reynaud (2018) for non-compliance in EU Fiscal Rules.

the repercussions of fiscal rule violations affect both society and the government, I argue that quantifying sanctions could be an important corrective mechanism for the central authorities or the fiscal council to enforce the fiscal rules in the SSA countries, especially when there exists a trade-off between fiscal flexibility and discipline.

Quantitatively, I demonstrate that, when not perfectly enforced, the optimal deficit limit relies on the marginal cost of sanctions[6]. If the penalty schedule is constant or infinite, then a sanction does not matter for the implementation of the optimal deficit rule. This shows that establishing an infinite or constant sanction leads the government to breach the limit at a maximum cost. As the government is assumed to be shortsighted, it will over-borrow to compensate for the maximum cost of breaching the rule. Since the society also bears the cost of the rule violation, it will tighten the deficit limit, and in fine, the limit is breached at no cost.[7] If the sanction is set to increase with the level of borrowing, then the optimal deficit limit is more relaxed.

The quantitative results show that the marginal cost of the deficit limit violation amplifies the level of the optimal deficit limit. Calibration for Sub-Saharan African (SSA) economies suggests that an optimal deficit limit ranges from 5.12% to 9.5% of GDP, while the sanctions vary between 0.33% and 2.7% of GDP. A significant insight from these findings is the imperative to reform the fiscal framework. This can be accomplished by enhancing the effectiveness of fiscal councils and introducing a fiscal rule that incorporates penalties as a corrective mechanism.

Related literature.– My paper delves into the body of literature that explores fiscal rules within contexts where governments lack commitment. First, This paper is related to the literature that studies the impact of lack of strong commitments on the design and effectiveness of fiscal policies (e.g., Chari and Kehoe (2007), Chari and Kehoe (2008), Cooper et al. (2008), Aguiar et al. (2015), Chari et al. (2016), Rodden (2002)). The main result of this literature is that fiscal rules are thought of as a solution to discipline a government facing limited commitment. My contribution to this literature is that I exogenize the set of policy instruments (sanctions) and I quantitatively evaluate the optimal deficit limit when a government faces the penalties that increase with the frequency of rule violation.

My paper builds also the works of Riley and Zeckhauser (1983), Athey, Atkeson, and

6. An optimal deficit is more relaxed when the frequency of its limit violation is high.
7. see Halal and Yared, 2018.

Kehoe (2005), as well as the recent work of Fuchs and Skrzypacz (2015) which concluded that the optimal solution is bang-bang incentives. In most of these papers, the authors employ the Lagrangian method to derive the optimal policy rules. Additionally, this literature features most of the time an economic environment that assumes perfect enforcement. I depart from this literature by introducing a linear assumption on the enforcement that jointly affects society and the government. Closely related papers are Amador, Werning, and Angeletos (2006), and Halac & Yared (2018, 2022), which study the design of optimal fiscal rule in an environment with full and limited enforcement. These authors rely on some distributional conditions and use perturbation arguments to characterize the optimal fiscal rule and derive the structure of the enforcement. I contribute to this literature by quantifying an optimal fiscal deficit limit that incorporates a linear penalty. Using a Lagrangian approach, I derive and evaluate the optimal fiscal deficit which depends on the distribution of the shocks and the marginal cost of the penalties - jointly determined by the society and the government.

Finally, this paper is related to the literature that analyzes hyperbolic discounting and the benefits of commitment instruments (e.g., Barro (1999), Bisin, Lizzeri, and Yariv (2013), Krusell, and Smith (2010), Krusell and Smith (2003), Laibson (1997), Lizzeri and Yariv (2013), and Phelps and Pollak (1968), Guillaume Sublet (2023), Lucien Chaffa (2023). I contribute to this literature by taking into account the stringency of the policy instruments and I show that it affects the optimal fiscal rule. In addition, I contribute to the literature methodically by taking the theoretical fiscal policy model to the data.

Layout.- The rest of the paper is organized as follows. In Section 2, I present the data and the empirical facts. Section 3 describes the model. I present the quantitative results in Section 4. Section 5 concludes.

2 Data sources and empirical evidence

This section provides a comprehensive overview of the empirical findings related to the primary balance, highlighting the frequency of noncompliance with current fiscal framework. It will also detail the various data sources utilized in this analysis. To begin, I will present the relevant data, including key statistics and trends in the primary balance

over time. Following this, I will show the motivational factors that underlie the observed pattern.

2.1 Data sources

In this section, I present the data derived from four sources of macroeconomic information. The first source consists of comprehensive data on government finance, which covers 202 countries from 1990 to 2022. This dataset includes details about the primary balance and fiscal balance. The second source provides insights into the use and design of fiscal rules, encompassing both national and supranational fiscal rules across 106 economies from 1985 to 2021. [8] In detail, the data contains four types of rules : Budget Balance Rules (BBR), Debt Rules (DR), Expenditure Rules (ER), and Revenue Rules (RR), applying to the central or general government or the public sector. The third data is aggregate governance indicators for over 200 countries over the period 1996 to 2022. In this database, I use the government's effectiveness as a proxy for the government's reputation. [9]. I supplement my datasets with the fiscal council database. This data describes the features of fiscal institutions across countries as of 2021.

Primary balance. I use the government finance database from the World Bank. [10] The *primary balance* is the difference between the amount of revenue a government collects and the amount it spends on providing public goods and services excluding net interest payments on public debt. A primary deficit occurs in a given year if a country spends more on public goods and services than it collects in taxes. [11] Put differently, a country that runs a primary deficit must borrow money to pay for the everyday public goods and services it provides.

Fiscal institution. I use fiscal institution and fiscal council interchangeably in this paper. I use the International Monetary Fund (IMF) fiscal council dataset. The data includes low-income, emerging, and advanced economies. A *fiscal council* is an institution

8. Fiscal rules set at the country level are called national fiscal rules and these set up at a regional or for economic union are supranational rules.

9. See Dovis and Kirpanali (2020) for the importance of government's reputation plays in explaining the fiscal rule noncompliance

10. see Kose et al. (2022) for more detailed information on this database.

11. The fiscal balance is the difference between a government's revenues and its expenditures

mandated to assess publicly and independently the government's fiscal policies. It takes 1 if there exists an institution that has characteristics such as formal guarantees of independence, accountability requirements, and human resources ; and it takes 0 otherwise.

Noncompliance. I use the Fiscal Rules Database by IMF which presents detailed information on types of fiscal rules, their legal basis, coverage, as well as enforcement procedures. The database contains indicators which are *de jure* measures. I use data on government finance to document whether a country complies or not with its existing rules. Put simply, *"noncompliance"* occurs if a country does not respect the numerical limit it has put on its budgetary aggregates. I define it as a dummy variable that takes 1 if a country complies with the limit of its fiscal rules and 0 otherwise.

Governance effectiveness index. I use data on the government effectiveness index from the World Bank's Worldwide Governance Indicators (WGI). [12] This index measures the quality and perceptions of public services, the degree of its independence from political pressures, and the government's commitments to the policies. It is between -2.5 and 2.5, where a score of 2.5 means the most effective government.

Overview on Fiscal Rules in SSA countries. In this paper, I quantitatively evaluate the Fiscal Rules in place in the Sub-Saharan African countries. The data contains a total of 24 SSA countries that have at least one fiscal rule in place. As of the end of 2021, 21 countries have a BBR. It is important to highlight that the definition of BBR is country-specific or monetary union-specific and varies across time. In 2000, the West African Economic and Monetary Union (WAEMU) which is constituted of 08 Member States (Benin, Burkina Faso, Côte d'Ivoire, Guinea-Bissau, Mali, Niger, Senegal, and Togo) implemented a fiscal union including a balanced budget rule (excluding budget grants and foreign-financed capital expenditures). However, in January 2015, WAEMU made some changes to its member countries' BBR and defined a new balanced budget (including grants) as a fiscal deficit below 3% of GDP. [13] The second fiscal union is the Central African Economic and Monetary Community (CEMAC). The member states of

12. For more information, see Daniel Kaufmann and Aart Kraay (2023). Worldwide Governance Indicators, 2023 Update (www.govindicators.org), Accessed on 01/19/2023

13. WAEMU has suspended its fiscal rules since April 2020.

CEMAC are Cameroon, the Central African Republic, Chad, the Republic of Congo, Equatorial Guinea, and Gabon. CEMAC commission established the BBR in 2002 and changed it in 2017 to a deficit rule that should remain below 1.5% of GDP. My sample is supplemented by countries from the East African Monetary Union (EAMU). The fiscal union from EAMU is constituted of Burundi, Kenya, Rwanda, South Sudan, Tanzania, and Uganda and contains a supranational rule that includes a ceiling on fiscal deficit of 3% of GDP since 2013.

2.2 Empirical evidence

I document three facts :

Fact 1 : There is a persistence in primary deficit in SSA countries. Figure 1 depicts the median of the primary balance in SSA countries from 1990 to 2019. The primary balance is the difference between general government revenues and expenditures net of interest payments on public debt. From 2000 to 2021, SSA countries had a persistent primary deficit, leading to a build-up of government debt over these periods. Meanwhile, most of them have implemented a Budget Balanced rule during the same period.

Fact 2 : The compliance with the fiscal rule is low and there is heterogeneity in fiscal institutions across countries. From 1965 to 2015, SSA countries did not comply with any fiscal rule an average of 30 times, which means non-compliance with fiscal rules is more frequent in SSA countries.[14] The table 1 reports the percentage of countries that implemented a BBR, their enforcement procedure, and legal basis. In SSA, 62% of the countries have *de jure* BBR, while none of these countries enforced it. Moreover, 75% of countries that have BBR enshrined it on a statutory basis. From 2000 to 2019, more than 65% has not complied with a deficit rule (Figure 3).

Fact 3 : Lower compliance is associated with more deficit. The literature on the effect of fiscal rules on the primary balance shed light on the fact that this effect could be mitigated (see Dovis and Kirpalani, 2020). Table 2 presents results from a regression of compliance with the rule on the primary deficit. The cross-product captures the marginal effect of non-compliance on the primary deficit during a period of low borrowing cost. All regressions are done with country and year fixed effect. The estimated coefficient

14. For more details on the fiscal rules violation, see Reuter (2017), and Caselli et al. (2018).

of compliance on the primary deficit is positive, which means that a country with low compliance with the deficit limit has a higher primary deficit.

I use these facts to discipline my model. In my model, the sanctions are endogenous, whereas the frequency of compliance is exogenous. I present the model in the next section.

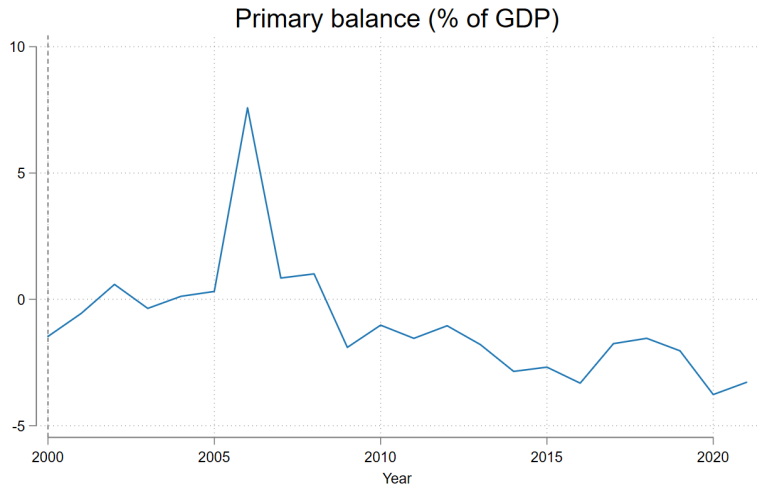

Figure 1 – *Governance finance in SSA countries*

Notes : This figure represents the median of primary balance as a percentage of GDP of SSA countries over the period 2000 and 2021. The countries are Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Côte d'Ivoire, Equatorial Guinea, Gabon, Guinea Bissau, Kenya, Liberia, Mali, Mauritius, Namibia, Niger, Nigeria, Rwanda, Senegal, South Sudan, Tanzania, Togo, Uganda. These countries have at least one fiscal rule in place as of the end of 2021.

Table 1 – *Enforcement of fiscal rules and legal basis (2021)*

	Type of enforcement		**Legal basis**
	De jure	**De facto**	**Statutory**
BBR	62%	0%	75%

Notes : This table showcases an intriguing combination of enforcement measures that operates on both national and supranational levels. The enforcement measure is indicated by a value of 1 if the country or region establishes a sanction or corrective measure to uphold the limit. The IMF fiscal rules datasets outline the concept of *De jure* enforcement. I compare the primary balance with the BBR limit established by each countries to derive the *De facto* enforcement. Regarding legal foundations, the strictest rules are established in the constitution, making modifications challenging.

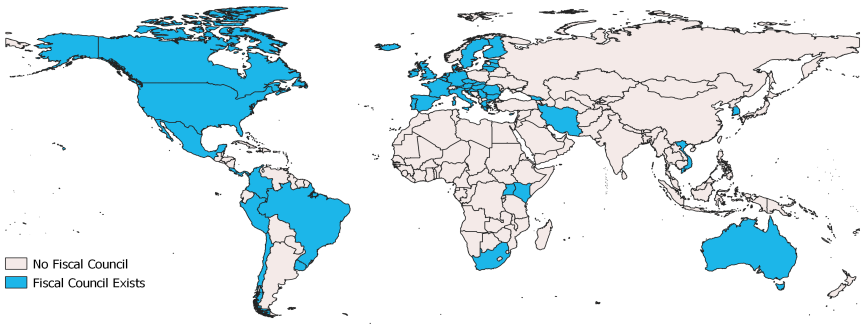

Figure 2 – *Fiscal institutions around the world, 2021*

Notes : In this figure, I represent the *De jure* measures of fiscal institutions around the world, reflecting their status and implementation as of the end of 2021.

Table 2 – *Regression of compliance on primary deficit*

Primary balance (% of GDP)	(1)	(2)
Compliance with the rule (CR)	5.640***	6.376***
Borrowing cost in the percentage of GDP (BRC)	−0.365**	−0.137
CRxBRC	0.460**	
Observations	820	820
R-squared	0.511	0.509
Number of countries	41	41

Notes : Regressions are done with country and year-fixed effects. CR is a dummy variable that takes the value 1 if a country complies with the rule in a given year. BRC is the value of the interest payments on public debts in the percentage of GDP. The data covers the period 2000-2019 and SSA countries. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

3 Model description

In this section, I lay down a comprehensive theoretical framework that underpins my analysis of a deficit rule characterized by imperfect enforcement mechanisms. In this paper, I build a simple and tractable fiscal policy model that features small open economies consisting of N states or countries and sanctions. The countries are indexed by $i \in \{1, 2, \ldots, N\}$. To evaluate the importance of enforcement in disciplining each country, I employ a model of fiscal policy similar to that analyzed by Amador, Werning,

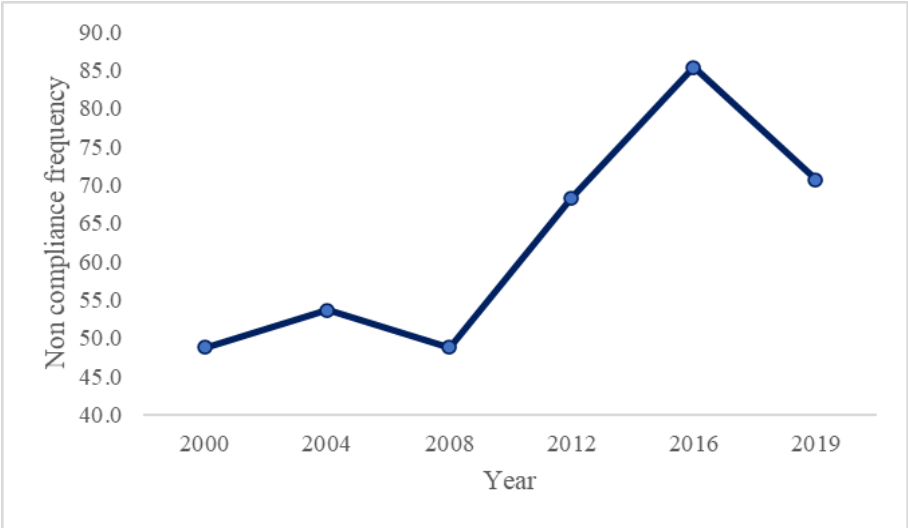

Figure 3 – *Frequency of deficit rule non-compliance*

and Angeletos (2006) and Halac and Yared (2022). The model is built on three key ingredients. The first ingredient comprises governments that exhibit present bias. These governments need discipline in their spending in the sense that they inherently put more value on their future valuation than their citizenry by prioritizing short-term fiscal needs and immediate benefits over long-term considerations. The second ingredient of the model is that each government faces idiosyncratic shocks to its fiscal needs when making spending and borrowing decisions. For instance, these shocks can manifest in various forms, such as sudden economic downturns, unexpected fluctuations in revenue, or unforeseen expenditures that arise due to natural disasters or public health crises. The shock to the economy is denoted by θ. The shock Θ is drawn from a distribution, denoted F, whose support Θ is bounded : $\Theta \equiv [\underline{\theta}, \bar{\theta}]$. The distribution F is continuously differentiable with probability density function $f(\theta) > 0$. The third ingredient evolves around the enforceability of the fiscal rule. This aspect demands careful consideration as the trade-off between flexibility and enforceability is less straightforward. On the one hand, the government tends to overspend when it faces unexpected economic shocks. These shocks can create fiscal pressure for increased public spending to stimulate the economy or provide necessary services, leading to potential budgetary excesses. Conversely, on the other hand, society imposes penalties on governments through various mechanisms to constrain them from over-borrowing.

Penalty in my framework[15].– Let denote by $P_i(b_i)$ the penalty schedule that is a function of the government i's borrowing b_i . By institutionalizing a penalty, society is aiming to reduce the frequency of fiscal rule violations. The cost of rule violation is faced by the government as well as by society. I assume then that the penalties are costly for both society and the government. To elaborate, I impose a linear restriction on the penalty schedule. This assumption implies that each country that violates frequently the rule will face high sanctions. Such a system of penalties aims to deter misconduct by ensuring that the sanctions are substantial enough to discourage repeated fiscal violations, ultimately fostering compliance and promoting a more orderly framework for governance. Given a level of borrowing b_i, a penalty for the government i is given by

$$P_i(b_i) = \gamma b_i, \tag{1}$$

where γ is the slope of the penalty schedule. A higher value for γ means high penalties. γ is assumed to be related to the frequency of fiscal rule violation, which means that a government that violates the rule more frequently should face harsher penalties and consequently have a high value for γ [16]. The sanction in my framework aims to disincentive countries that breach the threshold values frequently. The sanction in my model can be viewed as a preventive mechanism at the central authority's disposal to enforce the rule.

Government's Objective.– Each government i faces a shocks θ_i to its fiscal needs. A government i's welfare after the realization of the shock θ is given by

$$\theta_i U(g_i) + \delta_i \beta(V(b_i) - P_i(b_i)), \tag{2}$$

where $\delta_i \in (0, 1]$ is the present-bias, and $\beta \in (0, 1)$ is the discount factor. $V(.)$ denotes the utility function of future borrowing b_i ($b_i \in [\underline{b}, \bar{b}]$), and $\theta_i U(.)$ represents the utility function from a government $i's$ spending g_i - which is assumed to be positive - given the shock θ_i. I assume that both utilities $U(.)$ and $U(.)$ are strictly concave, strictly increasing, and twice continuously differentiable. Each government i faces a penalty schedule, $P_i(b)$, defined in equation (1). It is important to shed light on the fact the design of the fiscal

15. In this paper, I use penalty and sanction interchangeably.
16. γ defines the stringency of the sanctions.

rule in our setting does not depend on the shocks [17]. By multiplying the first-period utility by the taste of shocks θ, it embodies flexibility in the framework (Amador, Werning, and Angeletos (2006)). Each government is subject to a budget constraint

$$g_i = T_i + b_i, \tag{3}$$

where $T_i > 0$ represents a government $i's$ exogenous resources.

Social Welfare.– The social welfare yields the representation of the government's welfare before the realization of the shocks to its fiscal needs and it is given by

$$\mathbb{E}[\theta_i U(g_i) + \beta(V(b_i) - P_i(b_i))], \tag{4}$$

As $\delta_i < 1$, the social welfare (4) is different from the government's objective (2) in the sense that the government weighs the future more than the citizenry [18]. If the government is granted full flexibility, then it will face no penalty. By defining - $b_i^f(\theta_i)$- the level of borrowing conditional to the absence of penalty, a government i derives its policy by maximizing its welfare (2) subject to its budget constraint (3). Clearly, the allocation implemented by a government i in the absence of a penalty is given by

$$b_i^f(\theta_i) \in arg \max_{b_i \in [\underline{b}, \bar{b}]} \{\theta_i U(T_i + b_i) + \delta\beta V(b_i)\} \tag{5}$$

The optimality condition for equation (5) yields :

$$\theta_i U'(T_i + b_i^f(\theta_i)) = -\delta\beta V'(b_i^f(\theta_i)) \tag{6}$$

3.1 Design of the fiscal rule

Let us define $g_i^f(\theta_i)$ the government i's first-period spending when it faces shocks θ_i. For each government, i, its revenue T_i is assumed to not vary over time.

Fiscal rule. – Given a penalty schedule as defined in equation (1) and condition (6), I define a fiscal rule as a threshold θ_i^* such that if a government i faces shocks θ_i that is under the threshold ($\theta_i < \theta_i^*$), its borrowing level and spending is $b_i^f(\theta_i)$ and it pays no

17. For further documentation, see Piguilem and Schneider (2016).
18. this framework can be explained by a political turnover/uncertainty (Aguir and Amador (2011)).

penalty. If a government i faces shocks that is θ_i greater than the threshold ($\theta_i > \theta_i^*$), its borrowing level is $b_i^f(\theta_i^*)$ and the cost of the rule violation is $P_i(b_i^f(\theta_i^*)) = \gamma b_i^f(\theta_i^*)$. Under some conditions, this fiscal rule can be implemented using a maximum deficit limit (see Amador, Werning, and Angeletos (2006)). Formerly, the government i's allocation when facing economic shocks is defined as :

$$(b(\theta_i), P(\theta_i)) = \begin{cases} (b^f(\theta_i), 0) & \text{if } \theta_i < \theta_i^*, \\ (b^f(\theta_i^*), 0) & \text{if } \theta_i \in [\theta_i^*, \theta_i^{**}), \\ (b^p(\theta_i), P(b^p(\theta_i))) & \text{if } \theta_i > \theta_i^{**} \end{cases} \tag{7}$$

where $P_i(b_i^p(\theta_i)) = \gamma b^p(\theta_i)$.

Figure 4 depicts the design of the fiscal rule in my setting. The blue line denotes the fiscal rule in this framework (a maximum deficit limit). The red line in figure 4 represents the path when a government i has full flexibility ($b^f(\theta_i)$) until θ_i^{**}.

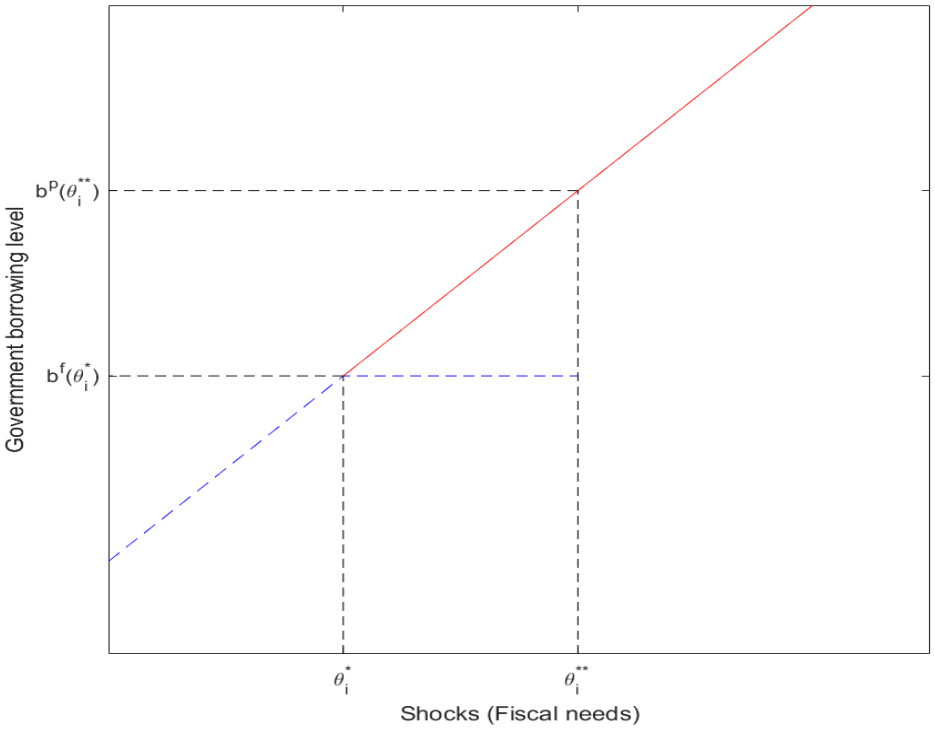

Figure 4 – *Design of a fiscal rule with penalty*

Notes : The blue dashed line depicts the path a government follows when it respects the flexible borrowing limit. The red line elicits the sanctions path that occurs when a government breaches the second borrowing limit.

By setting a fiscal rule, a government i pays sanctions when it breaches the limit (θ_i^*).

In other terms, when a government i follows the red line path on the figure 4 instead of respecting the deficit limit in place (blue line), then it faces a penalty that increases with the frequency of rule violation.

3.2 Optimal deficit limit rule with sanctions

I assume that each government i pays a penalty that increases with the frequency of non-compliance if it breaches the rule. This means that the more often a government breaches its regulations, the greater the financial repercussions it encounters. In this context, each government i chooses strategically its allocation so that it maximizes its expected welfare subject to the budget constraint (3), the penalty (1) it pays, and the condition (6). Consequently, the government's choices are influenced not only by the potential benefits of compliance but also by the sanctions associated to any noncompliance, leading to a careful balance between flexibility and enforceability.

A fiscal rule is *optimal* if the government solves

$$\max_{\theta_i^*,\theta_i^{**}} \left\{ \int_{\underline{\theta_i}}^{\overline{\theta_i}} \Big(\theta_i U(g_i(\theta_i)) + \beta \Big(V(b(\theta_i)) - P_i(b(\theta_i)) \Big) \Big) f(\theta_i) d\theta_i \right\} \quad (8)$$

subject to (1), (3), and (6).

Let us recall that if $\theta_i < \theta_i^*$, then a government i has full flexibility and can implement its allocation at no cost. By denoting by θ_{in} the threshold that satisfies the solution of the program (8), the first-order solution verifies

$$\int_{\theta_i^*}^{\overline{\theta_i}} \Big(\theta_i U'(g_i^f(\theta_{in}^*)) + \beta \Big(V'(b^f(\theta_{in}^*)) - P_i'(b^f(\theta_{in}^*)) \Big) \Big) f(\theta_i) d\theta_i = 0 \quad (9)$$

with $P_i'(b^f(\theta_{in}^*)$ the derivative of the penalty with respect to θ_i^*. The equation (9) shows how the average distortion above the threshold θ_{in}^* depends on the cost of rule violation. Using the conditions (1), (3), and (6), I show that the *optimal* fiscal rule is a threshold θ_{in}^* that verifies

$$\frac{\mathbb{E}[\theta_i | \theta_i \geq \theta_{in}^{**}]}{\theta_{in}^{**}} = \frac{1}{\delta} + \underbrace{\frac{\beta P_i'(b^p(\theta_{in}^{**}))}{\theta_{in}^{**} U'(g_i(\theta_{in}^{**}))}}_{\text{Marginal cost of punishment}} \quad (10)$$

The term $\frac{\beta P_i^{'}(b^f(\theta_{in}^{**}))}{\theta_{in}^{**}U'(g_i^f(\theta_{in}^{**}))}$, which I call the *marginal cost of punishment* captures the effects that the sanctions have on the allocation. These effects are positive. The sanctions add a new term to the *optimal* fiscal rule equation (10), which is the marginal cost of punishment. The effects of the marginal cost of punishment depend on the slope [19] of the penalty. I now show how the slope of the penalty schedule affects the *optimal* fiscal rule.

Claim 1 : If the penalty schedule is infinite, then the sanctions do not matter for the *optimal* fiscal rule. The intuition behind this claim is that imposing an infinite penalty allows the government to breach the rule at a maximum cost. As the government is assumed to be shortsighted, it will overborrow to compensate for the maximum cost of rule violation. In fine, the fiscal rule is breached at no cost, and society needs to tighten the deficit limit (Halac and Yared (2018)). In this case, the cost of constraining a government i to a threshold θ_{in}^{*} is equal to the benefit of adopting a fiscal rule.

Claim 2 : If the slope (severity) of the penalty increases, then the *optimal* fiscal rule increases. Intuitively, by setting a more severe penalty, society and the government pay a harsh cost when the rule is breached. Therefore, the fiscal rule needs to be relaxed when the sanctions are severe.

Given the linear penalty schedule, claim 2 shows that what matters is the severity (slope) of the penalty that society imposes on the government. Halac and Yared (2022) have shown that harsher penalties are too costly to society as well as to the government. A severe penalty benefits society when the deficit limit is more relaxed. In my framework, I assume a linear penalty schedule and find that the marginal cost of punishment does affect the *optimal* fiscal rule. In the next section, I present the quantitative analysis.

4 Quantifying the optimal deficit limit when enforcement is not perfect

In this section, I present the model's quantitative results. To begin with, I will present the calibration process, outlining the methods employed to ensure that the model accurately reflects the underlying data. Following this, I will discuss the specific results obtained from the calibration, highlighting key parameters and their implications. Finally, I will

19. Let us recall that I assume a linear penalty schedule if a government breaches the rule.

analyze the potential advantages of implementing a deficit rule accompanied by a sanction, focusing on how this approach can enhance fiscal discipline.

4.1 Calibration

The calibration uses government finance data from height (08) SSA countries : Côte d'Ivoire, Benin, Burkina Faso, Guinea-Bissau, Mali, Niger, Senegal, and Togo.

Fiscal needs shocks in my model.- For each country i, I assume a Constant Absolute Risk Aversion (CARA) form of the utility function : $U(g_i) = 1 - e^{-\alpha g_i}$. By assuming a CARA utility function, the shocks on a government i's fiscal needs (θ_i) can be interpreted as shocks on its revenues [20]. The CARA utility function offers a tractable way to map fiscal needs shocks to government revenue shocks.

The mapping between the shocks to fiscal needs(θ) and the government revenue is obtained as follows : $\theta_i = e^{-\alpha T_i}$, where T_i is the shock on government i's exogenous revenue. To obtain the distribution of the preference shocks of each country, θ_i, I use the times series data of each government i's revenue and map the distribution using a non-parametric approach. This method allows me to derive the preference shocks without making assumptions about their underlying distribution. It provides a clearer picture of how revenue varies across different governments over the observed periods. Let us define $f_T(.)$ and $f_\theta(.)$ respectively the density functions of the shock T_i and θ. The equation 11 establishes a functional link between fiscal needs shocks and shocks on government revenue :

$$f_\theta(x) = \frac{1}{\alpha x} f_T(-\frac{1}{\alpha} ln(x)) \tag{11}$$

I set the absolute risk aversion coefficient $\alpha = 1$ in my calibration. This choice is guided by existing research that suggests governments with weak institutions and reputations exhibit a significant tendency toward present bias—prioritizing immediate benefits over long-term gains (Halac and Yared, 2018).

Degree of present bias.- To determine the degree of present bias in my calibration, I assume that the average spending of each government i in the model matches the average spending observed in the data. In line with the literature, I set externally the discount

20. See Section 5.4 in Amador, Werning, and Angeletos (2006)

factor parameter of $\beta = 0.96$, suggesting a strong preference for current over future benefits.

Penalties in my model.- In my model, the penalties are designed as incentives for the government to comply with the rule. Theoretically, if the penalties are high enough, a government should adhere to avoid punishment. To apply the model to data, I make additional assumptions about the penalties, claiming that societies impose sanctions exogenously. Each country will breach its rules when faced with sufficiently high fiscal shock needs. In this context, the penalties will be set externally by society following the equation 12 :

$$P(b_i) = max(V(b_i)) - min(V(b_i)). \tag{12}$$

The equation 12 states that the penalties assigned to a country are determined by the maximum cumulative sanctions that can be imposed during a specified period of time. I calibrate the maximum possible penalties for each country by employing the equation 12 in an exogenous manner. This process involves considering conditions specific to each country, allowing for a tailored calibration of penalties that reflect their unique circumstances. As a result, the marginal cost of punishment is ultimately externalized within the framework of the model, leading to a marginal cost of zero in equation 10. With no penalties, the optimal flexible level of deficit limit using CARA utilities is given by the following equation 13 :

$$b^f(\theta) = \frac{1}{2\alpha} log(\frac{\theta}{\beta\delta}) \tag{13}$$

With θ representing the government fiscal needs, which have been calibrated using a non-parametric approach.

4.2 Discussions of the quantitative results

In this section, I discuss the quantitative results of the computation of optimal deficit limits for the height (08) SSA economies. This discussion aims to comprehensively understand how optimal deficit limits can be established.

The graph 5 displays the maximum penalties faced by each country in my sample. The variation among countries highlights the diverse approaches to fiscal enforcement, potentially shaped by their economic resilience, fiscal governance capabilities, and their

fiscal needs shocks. The penalty rates vary significantly across countries. Countries such as Togo have the highest penalties at 2.7% of GDP, signaling a more stringent fiscal policy framework to discipline its government spending. On the lower end, Burkina Faso faces penalties of 0.33%, which may indicate a different economic condition that requires flexibility in their fiscal framework.

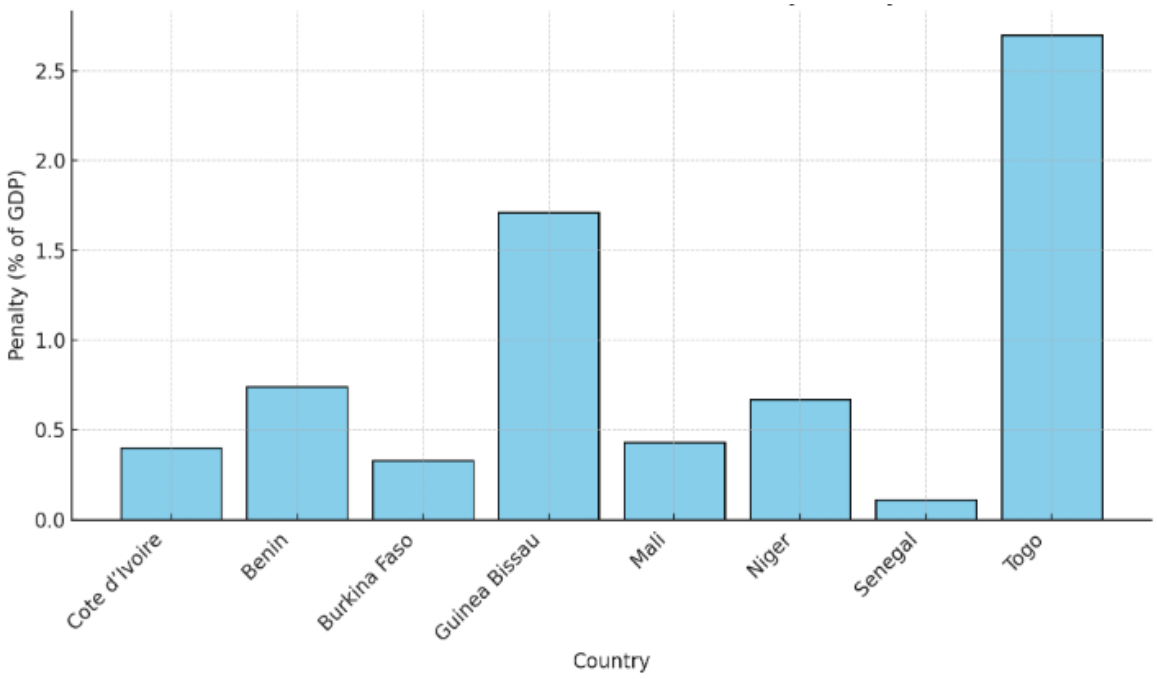

Figure 5 – *Maximum penalties (% of GDP) by country*

Notes : The bar chart presented above illustrates the maximum penalties that each country faced, measured as a percentage of their Gross Domestic Product (GDP). This analysis encompasses data collected over an extensive time-frame from 1960 to 1999, a period marked by the absence of fiscal rules in these nations. The sample of countries includes Côte d'Ivoire, Benin, Burkina Faso, Guinea-Bissau, Mali, Niger, Senegal, and Togo.

I conduct a quantitative assessment of the optimal deficit limits for each country, particularly where enforcement capabilities are limited. The optimal deficit limits, illustrated in the graph 6, reflect the distinct fiscal policies and economic contexts of each nation. These limits display a wide range, extending from the most fiscally conservative to the most flexible, which indicates a variety of approaches to fiscal management.

Burkina Faso has the highest optimal deficit limit at 9.5%, suggesting a greater degree of flexibility in its fiscal framework. In contrast, Côte d'Ivoire has a more stringent limit of 5.12%, likely shaped by its developmental priorities and public investment needs.

The differences in deficit limits among these countries highlight the importance of customized policies that address unique economic challenges. Policymakers should strive

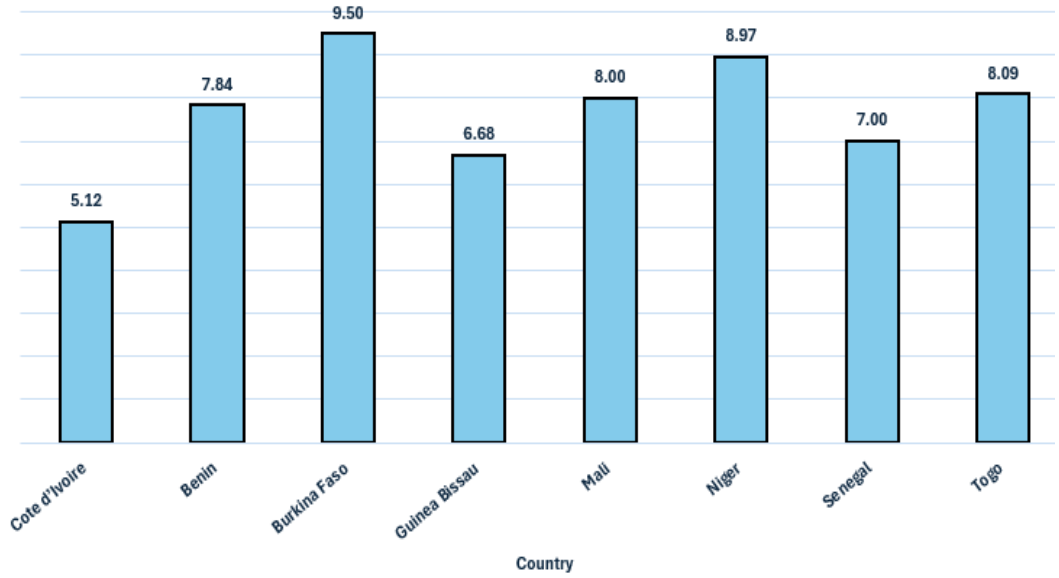

Figure 6 – *Optimal deficit limit (% of GDP) by country*

Notes : The bar chart presented above illustrates the optimal deficit limit for each country, where each country faces fiscal pressures and in an environment with limited enforcement.

to align these limits with their economic goals, ensuring they support sustainable development while preserving macroeconomic stability and effectively enforcing regulations.

5 Conclusion

In this paper, I present a tractable fiscal policy model that quantifies an optimal deficit limit with a sanction which depends on the level of borrowing. Put simply, a sanction in my model increases with the frequency of noncompliance such that a country that overborrows will face a harsh sanction. In fact, sanctions aim at disciplining the government from overspending when it faces economic shocks and is present-biased.

The extent to which the sanctions affect the *optimal* fiscal rule depends on the slope (severity) of the penalty. The *optimal* deficit limit solution shows that the deficit limit is more relaxed when society imposes a linear penalty on the government when it breaches the rule than when the deficit limit is breached at no cost.

Calibration for Sub-Saharan African (SSA) economies suggests that an optimal deficit limit ranges from 5.12% to 9.5% of GDP, while the sanctions vary between 0.33% and 2.7% of GDP. Subsequently, these findings underscore the need of reforming the existing

fiscal rules to enhance their effectiveness, as well as and strengthening the enforcement mechanism that ensures compliance with these rules in these countries.

Finally, the model I employ in this paper can be designed to quantify the *optimal fiscal rule* for any country that breaches its *fiscal guidelines*, particularly in instances where enforcement is inadequate. For example, this model could be utilized to quantify the *optimal deficit limit* for European Union countries, which have consistently violated the *Excessive Deficit Procedure (EDP)* without facing any repercussions. [21]

21. For additional insights into noncompliance with the EDP in the EU, refer to Guillaume Sublet (2023).

References

[1] Amador, M., Werning, I. and Angeletos, G.-M. (2006). Commitment vs. Flexibility. Econometrica, 74 : 365-396

[2] Azzimonti, M., Battaglini, M., and Coate, S. (2016). The Costs and Benefits of Balanced Budget Rules : Lessons from a Political Economy Model of Fiscal Policy, Journal of Public Economics, 136 : 45-61

[3] Chari, V. V., and Kehoe, P. (2008). Time Inconsistency and Free-Riding in a Monetary Union. Journal of Money, Credit, and Banking, 40 : 1329-1355

[4] Davoodi et al. (2022). Fiscal rules and fiscal councils : Recent trends and performance during the COVID-19 pandemic. IMF Working Paper

[5] Dessus, S., Diaz-Sanchez, J. L., and Varoudakis, A. (2016). Fiscal rules and the procyclicality of public investment in the West African economic and monetary union. Journal of International Development, 28(6), 887–901.

[6] Diamond, P. (1998). Optimal Income Taxation : An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates. American Economic Review, 88 : 83-95

[7] Diaz Kalan, F, M Popescu and J Reynaud (2018), "Thou shalt not breach : The impact on sovereign spreads of noncomplying with the EU fiscal rules", IMF working paper 18(87)

[8] Dovis, A. and Kirpalani, R. (2021). Rules without Commitment : Reputation and Incentives. *forthcoming* Review of Economic Studies

[9] Dovis, A., and Kirpalani, R. (2020). Fiscal Rules, Bailouts, and Reputation in Federal Governments. American Economic Review, 110 (3) : 860-88

[10] Guillaume Sublet (2023), The Optimal Degree of Discretion in Fiscal Policy, Revise & Resubmit Econometrica

[11] Halac, M., and Yared, P. (2020a). Inflation Targeting under Political Pressure. in Independence, Credibility, and Communication of Central Banking,

edited by Ernesto Pasten and RicardoReis, Santiago, Chile : Central Bank of Chile, 7-27

[12] Halac, M., and Yared, P. (2020b). Commitment vs. Flexibility with Costly Verification. Journal of Political Economy, 128 : 4523-4573

[13] Halac, M., and Yared, P. (2022a). Fiscal Rules and Discretion under Limited Enforcement. Econometrica

[14] Halac, M., and Yared, P. (2022b). Instrument vs. Target-Based Rules. Review of Economic Studies, 89 : 312-345

[15] Halac, M., and Yared, P. (2022c). A Theory of Fiscal Responsibility and Irresponsibility. working paper

[16] Halac, M., and Yared, P. (2018). Fiscal Rules and Discretion in a World Economy. American Economic Review, 108 : 2305-2334

[17] Halac, M., and Yared, P. (2014). Fiscal Rules and Discretion under Persistent Shocks. Econometrica, 82 : 1557-1614

[18] Kocherlakota, N. (2016). Rules versus Discretion : A Reconsideration. Brookings Papers on Economic Activity, 1-36

[19] Kose, M. Ayhan, Sergio Kurlat, Franziska Ohnsorge, and Naotaka Sugawara. 2022. "A Cross-Country Database of Fiscal Space." Journal of International Money and Finance 128 (November) : 102682.

[20] Lucien Chaffa (2023).Quantitative Exploration of Fiscal Rules for WAEMU Countries. Working paper.

[21] Yared, P. (2019). Rising Government Debt : Causes and Solutions for a Decades-Old Trend. Journal of Economic Perspectives, 33 : 115-140

A Proofs of main results

A.1 Proof of equation 10

Proof of equation 10. Each government maximizes its expected welfare subject to the budget constraint (3), the penalty (1), and it chooses its allocation (6).

$$\max_{\theta_i^* \in [\underline{\theta_i}, \overline{\theta_i}]} \left\{ \int_{\underline{\theta_i}}^{\theta_i^*} \left(\theta_i U(g_i^f(\theta_i)) + \beta \Big(V(b^f(\theta_i)) - P_i(b^f(\theta_i)) \Big) \right) f(\theta_i) d\theta_i \right.$$
$$\left. + \int_{\theta_i^*}^{\overline{\theta_i}} \left(\theta_i U(g_i^f(\theta_i^*)) + \beta \Big(V(b^f(\theta_i^*)) - P_i(b^f(\theta_i^*)) \Big) \right) f(\theta_i) d\theta_i \right\}$$

subject to :

$$P_i(b_i) = \gamma b_i$$

$$g_i = T_i + b_i$$

$$\theta_i U'(T_i + b_i^f(\theta_i)) = -\delta\beta V'(b_i^f(\theta_i))$$

The optimal solution of this program yields the following equation :

$$\int_{\theta_i^*}^{\overline{\theta_i}} \left(\theta_i U'(g_i^f(\theta_{in}^*)) + \beta \Big(V'(b^f(\theta_{in}^*)) - P_i^{'}(b^f(\theta_{in}^*)) \Big) \right) f(\theta_i) d\theta_i = 0$$

After substituting the condition (6) into the above equation, I obtain the following equation :

$$\int_{\theta_i^*}^{\overline{\theta_i}} \left(\theta_i U'(g_i^f(\theta_{in}^*)) + \beta \Big(-\frac{1}{\beta\delta} \theta_{in}^* U'(T_i + b_i^f(\theta_{in}^*)) - P_i^{'}(b^f(\theta_{in}^*)) \Big) \right) f(\theta_i) d\theta_i = 0$$

By rewriting the equation above, it gives :

$$\int_{\theta_i^*}^{\overline{\theta_i}} \theta_i U'(g_i^f(\theta_{in}^*)) f(\theta_i) d\theta_i = \frac{1}{\delta} \theta_{in}^* \int_{\theta_i^*}^{\overline{\theta_i}} U'(T_i + b_i^f(\theta_{in}^*)) f(\theta_i) d\theta_i + \beta P_i^{'}(b^f(\theta_{in}^*)) \int_{\theta_i^*}^{\overline{\theta_i}} f(\theta_i) d\theta_i \tag{14}$$

After dividing the right-hand and left-hand sides of equation (14) by $\theta^*_{in}\int_{\theta^*_i}^{\overline{\theta_i}} U'(T_i + b^f_i(\theta^*_{in}))f(\theta_i)d\theta_i$, I obtain :

$$\frac{\int_{\theta^*_i}^{\overline{\theta_i}} \theta_i U'(g^f_i(\theta^*_{in}))f(\theta_i)d\theta_i}{\theta^*_{in}\int_{\theta^*_i}^{\overline{\theta_i}} U'(T_i + b^f_i(\theta^*_{in}))f(\theta_i)d\theta_i} = \frac{1}{\delta} + \frac{\beta P_i'(b^f(\theta^*_{in}))\int_{\theta^*_i}^{\overline{\theta_i}} f(\theta_i)d\theta_i}{\theta^*_{in}\int_{\theta^*_i}^{\overline{\theta_i}} U'(T_i + b^f_i(\theta^*_{in}))f(\theta_i)d\theta_i}$$

With a little algebra, the *optimal fiscal rule* is given by

$$\frac{\mathbb{E}[\theta|\theta \geq \theta^*_{in}]}{\theta^*_{in}} = \frac{1}{\delta} + \underbrace{\frac{\beta P_i'(b^f(\theta^*_{in}))}{\theta^*_{in}U'(g^f_i(\theta^*_{in}))}}_{\text{Marginal cost of punishment}} \tag{15}$$

B Heterogeneity in government fiscal needs

In this section, I present evidence of heterogeneity in governance fiscal needs across SSA countries. The graphs 7 and 8 elicit the heterogeneity in these countries' spending and revenue between 2000 and 2022. This indicates that, whilst some of these countries are in fiscal union, their fiscal needs seem to diverge. I run Analysis of Variance (ANOVA) regression to test whether these countries are different or not in the means of their fiscal needs and the pvalue of the models is less than 1% . This indicates that they are statistically different in the means of their fiscal needs.

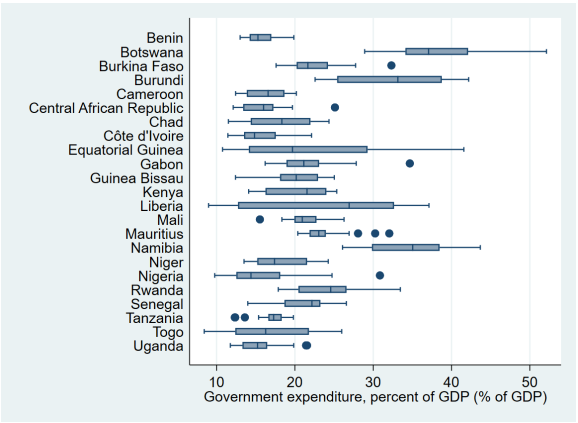

Figure 7 – *Government spending (% of GDP) from 2000 to 2022*

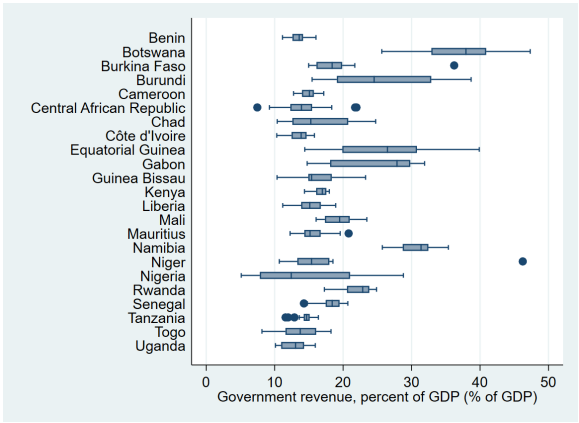

Figure 8 – *Government revenue (% of GDP) from 2000 to 2022*

Notes : The graphs 7 and 8 depict respectively the box plot of each SSA countries spending and revenue, as percentage of GDP between 2000 and 2022.The size of the box plots are uneven. A larger size means more volatility across time in a country fiscal needs.

Table 3 – *Descriptive statistics of government spending as percentage of GDP from 2000 to 2022*

Country	**mean(expgdp)**	**sd(expgdp)**	**med(expgdp)**
Benin	15.71	2.02	15.28
Botswana	38.07	5.41	37.07
Burkina Faso	22.29	3.29	21.65
Burundi	32.44	6.76	33.14
Cameroon	16.30	2.56	16.58
Central African Republic	15.74	2.92	16.00
Chad	18.13	3.86	18.34
Côte d'Ivoire	15.68	2.78	14.84
Equatorial Guinea	22.17	9.10	19.69
Gabon	21.55	4.06	21.13
Guinea Bissau	19.84	3.48	20.17
Kenya	20.54	3.82	21.54
Liberia	24.17	9.66	26.93
Mali	21.28	2.40	20.94
Mauritius	24.17	9.66	26.93
Namibia	34.49	5.38	35.04
Niger	18.21	3.58	17.40
Nigeria	16.03	4.91	14.39
Rwanda	24.12	4.28	24.57
Senegal	21.42	3.44	22.17
Tanzania	17.10	2.05	17.29
Togo	16.64	5.08	16.27
Uganda	15.59	2.63	15.23

Notes : The variable *expgdp* is the government spending as percent of GDP, and the variables *sd* and *med* stand respectively for a standard deviation and a median. The table depicts descriptive statistics of 23 SSA countries that have at least one fiscal rule in place as of end of 2021 as I drop South Sudan. South Sudan implemented a fiscal rule in recently in 2013.

Table 4 – *Descriptive statistics of government revenue as percentage of GDP from 2000 to 2022*

Country	**mean(revgdp)**	**sd(revgdp)**	**med(revgdp)**
Benin	13.37	1.12	13.58
Botswana	36.95	5.86	37.94
Burkina Faso	18.92	4.26	18.40
Burundi	25.71	7.60	24.53
Cameroon	14.87	1.13	15.08
Central African Republic	14.29	3.34	13.92
Chad	16.72	4.54	15.28
Côte d'Ivoire	13.64	1.38	13.87
Equatorial Guinea	25.76	7.03	26.47
Gabon	25.10	5.90	27.87
Guinea Bissau	16.20	3.42	15.40
Kenya	16.67	1.11	16.96
Liberia	15.32	2.11	15.13
Mali	19.29	2.06	19.49
Mauritius	15.73	2.32	15.17
Namibia	30.92	2.55	31.39
Niger	16.56	6.93	15.39
Nigeria	14.70	7.33	12.42
Rwanda	21.99	2.36	22.87
Senegal	18.26	1.70	18.41
Tanzania	14.52	1.16	14.67
Togo	13.71	3.03	13.73
Uganda	12.90	1.78	13.06

Notes : This table displays descriptive statistics of 23 SSA countries that have at least one fiscal rule in place as of end of 2021. I drop South Sudan from as it has implemented a fiscal rule in 2013. The variable *revgdp* is the government revenue as percent of GDP, and the variables *sd* and *med* represents respectively the standard deviation and the median.